

Forward probabilities

(1) $V_i \rightarrow V_j V_k$
 $V_i \rightarrow w_j$

(2) Outside probability $\alpha_j(p, q) = p(w_{1(p-1)}, V_{pq}^j, w_{(q+1)m} | G)$
 Inside probability $\beta_j(p, q) = p(w_{pq} | V_{pq}^j, G)$

(3) Base case: $\beta_j(k, k) = p(w_k | V_{kk}^j, G)$
 $= p(V^j \rightarrow w_k | G)$

(4) Induction: $\beta_j(p, q) = p(w_{pq} | V_{pq}^j, G)$
 $= \sum_{r,s} \sum_{d=p}^{q-1} p(V^j \rightarrow V^r V^s) \beta_r(p, d) \beta_s(d+1, q)$

(5) $S \rightarrow NP VP$ 1.0 $NP \rightarrow NP PP$ 0.4
 $PP \rightarrow P NP$ 1.0 $NP \rightarrow astronomers$ 0.1
 $VP \rightarrow V NP$ 0.7 $NP \rightarrow ears$ 0.18
 $VP \rightarrow VP PP$ 0.3 $NP \rightarrow saw$ 0.04
 $P \rightarrow with$ 1.0 $NP \rightarrow stars$ 0.18
 $V \rightarrow saw$ 1.0 $NP \rightarrow telescopes$ 0.1

(6)

	1	2	3	4	5
1	$\beta_{NP} = \mathbf{0.1}$		$\beta_S = 0.0126$		$\beta_S = 0.0015$
2		$\beta_{NP} = \mathbf{0.04}$ $\beta_V = \mathbf{1.0}$	$\beta_{VP} = \mathbf{0.126}$		$\beta_{VP} = 0.0158$
3			$\beta_{NP} = \mathbf{0.18}$		$\beta_{NP} = 0.0129$
4				$\beta_P = \mathbf{1.0}$	$\beta_{PP} = \mathbf{0.18}$
5					$\beta_{NP} = \mathbf{0.18}$
	<i>astr-ers</i>	<i>saw</i>	<i>stars</i>	<i>with</i>	<i>ears</i>